BMP 2009
Mathe 2

1) a)

$$\frac{\frac{(x+1)^2}{x^2-1}}{\frac{1}{x+1}-\frac{1-x^2}{(x-1)^2}} = \frac{\frac{(x+1)^2}{(x+1)(x-1)}}{\frac{1}{x+1}-\frac{(1+x)(1-x)}{(x-1)(x-1)}}$$

$$= \frac{\frac{(x+1)^2}{(x+1)(x-1)}}{\frac{1}{x+1}+\frac{(1+x)\cancel{(1-x)}}{\cancel{(1-x)}(x-1)}} = \frac{\frac{(x+1)^2}{(x+1)(x-1)}}{\frac{(x-1)+(1+x)(x+1)}{(x+1)(x-1)}}$$

$$= \frac{(x+1)^2 \cdot \cancel{(x+1)(x-1)}}{\cancel{(x+1)(x-1)}(x-1+x^2+2x+1)} = \underline{\underline{\frac{(x+1)^2}{x(x+3)}}}$$

b)

$$\frac{\frac{4}{a^2}-\frac{4}{ab}+\frac{1}{b^2}}{\frac{1}{a}-\frac{1}{2b}} = \frac{\frac{4b^2-4ab+a^2}{a^2b^2}}{\frac{2b-a}{2ab}}$$

$$= \frac{(2b-a)^2 \cdot 2\cancel{ab}}{a^2b^2\,\cancel{(2b-a)}} = \underline{\underline{\frac{2(2b-a)}{ab}}}$$

c)

$$\left[(xy^{-2})^{-\frac{1}{2}} \cdot (x^{-\frac{3}{2}}y) \cdot (x^{-1})^{-\frac{2}{3}}\right]^3$$

$$= (xy^{-2})^{-\frac{3}{2}} \cdot (x^{-\frac{3}{2}}y)^3 \cdot (x^{-1})^{-2}$$

$$= x^{-\frac{3}{2}}y^3 \cdot x^{-\frac{9}{2}}y^3 \cdot x^2 = x^{-4}y^6 = \underline{\underline{\frac{y^6}{x^4}}}$$

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2) $\frac{-\sqrt{x-1}}{x-1} + \sqrt{6+x} = \frac{11}{\sqrt{x-1}} \quad \Big| \cdot (x-1)$

$-\sqrt{x-1} + \sqrt{6+x}\,(x-1) = 11\sqrt{x-1}$

$-\sqrt{x-1} + \sqrt{6+x}\,(\sqrt{x-1})^2 = 11\sqrt{x-1} \quad \Big| : \sqrt{x-1}$

$-1 + \sqrt{6+x}\sqrt{x-1} = 11$

$\sqrt{(6+x)(x-1)} = 12$

$x^2+5x-6 = 144$

$x^2+5x-150 = 0$

$x_1 = 10 \quad , \quad x_2 = -15$

Def.-Bereich: $x > 1$, aus $\sqrt{x-1}$

$\underline{\underline{D = \{ x \in \mathbb{R} \mid x > 1 \}}}$ ✓

$\underline{\underline{\mathbb{L} = \{ 10 \}}}$ ✓

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3) geg: $y = ax^2 - 2x - 4$, $y = 2x + 3$

ges: a) a

b) P

a) $ax^2 - 2x - 4 = 2x + 3$

$ax^2 - 4x - 7 = 0$

$D = b^2 - 4ac \Rightarrow D = (-4)^2 + 28a = 0$

$28a = -16$

$a = \frac{-16}{28} = -\frac{4}{7}$

$\Rightarrow \quad \underline{\underline{y = -\frac{4}{7}x^2 - 2x - 4}}$ ✓

b) Berührungspunkt P

$$-\frac{4}{7}x^2 - 2x - 4 = 2x + 3$$

$$-\frac{4}{7}x^2 - 4x - 7 = 0 \quad | \cdot 7$$

$$-4x^2 - 28x - 49 = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{28 \pm \sqrt{\underbrace{28^2 - 16 \cdot 49}_{= 0}}}{-8}$$

$$= -\frac{28}{8} = -\frac{7}{2}$$

y - Koord. : $y = 2x + 3 = 2\left(-\frac{7}{2}\right) + 3 = -4$

$$\Rightarrow \underline{\underline{P\left(-\frac{7}{2} \middle| -4\right)}}$$ 2

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4)

$$\left| \begin{array}{l} \sqrt{a-5} + \sqrt{b+2} = 5 \\ \sqrt{a+b} = 4 \end{array} \right| \Rightarrow a + b = 16$$

$$a = 16 - b$$

$$\sqrt{16 - b - 5} + \sqrt{b+2} = 5$$

$$\sqrt{11 - b} + \sqrt{b+2} = 5 \quad |^2$$

$$11 - b + 2\sqrt{11-b}\sqrt{b+2} + b + 2 = 25$$

$$2\sqrt{(11-b)(b+2)} = 12 \quad |:2 \; |^2$$

$$(11-b)(b+2) = 36$$

$$11b + 22 - b^2 - 2b = 36$$

$$-b^2 + 9b - 14 = 0 \quad (\leftarrow 2P)$$

$b_1 = 2, \; b_2 = 7$

$a_1 = 14, \; a_2 = 9$ $\quad \underline{\underline{L = \{(14/2), (9/7)\}}}$ 3

5) a) $\ln(x^2) \cdot \ln(x) = \frac{1}{2}$

$2\ln(x) \cdot \ln(x) = \frac{1}{2}$

$2(\ln(x))^2 = \frac{1}{2}$

$(\ln(x))^2 = \frac{1}{4}$

$\Rightarrow \ln(x_1) = +\frac{1}{2}\ ,\quad \ln(x_2) = -\frac{1}{2}$

$\Rightarrow x_1 = e^{+\frac{1}{2}} = \sqrt{e}\ ,\quad x_2 = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

b) $a^{x+1} - a^{x-1} = b^x$

$a \cdot a^x - \frac{a^x}{a} = b^x \quad | \cdot a$

$a^2 a^x - a^x = a b^x$

$a^x(a^2-1) = a b^x$

$\frac{a^x}{b^x} = \frac{a}{a^2-1}$

$x = \frac{\log\left(\frac{a}{a^2-1}\right)}{\log\left(\frac{a}{b}\right)}$

c) $\ln(11x-10) + (\ln(11x-10))^2 = 6$

$y = \ln(11x-10)$

$y^2 + y - 6 = 0 \Rightarrow y_1 = 2$

$y_2 = -3$

$\ln(11x-10) = 2$

$11x - 10 = e^2$

$x_1 = \frac{e^2+10}{11}$

$\ln(11x-10) = -3$

$11x - 10 = e^{-3}$

$x_2 = \frac{e^{-3}+10}{11}$

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6) gg: $y = 2x^3 - 3x^2 - 3x + 2$

gs: a) Skizze

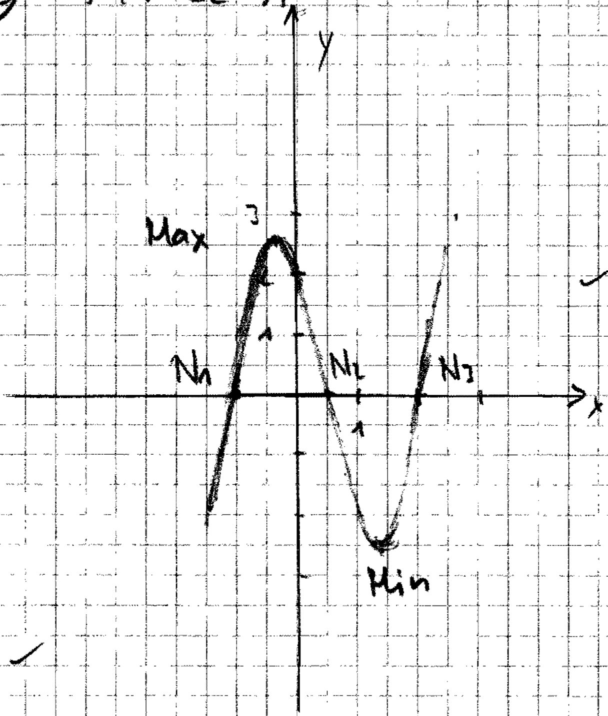
b) Fläche A

a)

$N_1(-1/0)$

$N_2(1/2\,/\,0)$

$N_3(2/0)$



Minimum

$(1{,}37/-2{,}60)$

Maximum

$(-0{,}37/2{,}60)$

b) Fläche A

Dreieck $A_\Delta = \frac{\Delta N \cdot y_{max}}{2} = \frac{(0{,}5-(-1))\cdot 2{,}6}{2}$

$= 1{,}95$

$A_\Delta \approx 2$

$\Rightarrow$ Fläche total: ≈ 4

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genau: $\int_{-1}^{1/2} (2x^3 - 3x^2 - 3x + 2)\,dx = 2{,}53$

$A \approx 5$

7) Viren

$A_1 = 2h \longrightarrow N_1 = 8000$

$A_2 = 3h \longrightarrow N_2 = 27'000$

$\tau = \Delta A = 1h \rightarrow a = \frac{N_2}{N_1} = 3,375$

$N(A) = N_0 \cdot a^{\frac{A}{\tau}}$

$= N_0 \cdot 3,375^{\frac{A}{1}}$

$\Rightarrow N_0 = \frac{N(A)}{3,375^A} = \frac{8000}{3,375^2} = 702,3$

$\Rightarrow N(A) = N_0 \cdot 3,375^A$

a) für $A = 24h$

$\underline{\underline{N(A) = 3,35 \cdot 10^{15} \text{ Viren}}}$

b) $N(A) = N_0 \cdot a^A \Rightarrow A = \frac{\log\left(\frac{N(A)}{N_0}\right)}{\log a} = \underline{\underline{2,52h}}$ 2

8) geg: $A(1/9)$, $B(2/16)$, $C(0/0)$

$C \in p \Rightarrow c = 0$

$y = ax^2 + bx$

$$\left| \begin{array}{l} 9 = a(1)^2 + b(1) \\ 16 = a(2)^2 + b(2) \end{array} \right|$$

$$\left| \begin{array}{l} 9 = a + b \\ 16 = 4a + 2b \end{array} \right| \qquad \begin{array}{l} a = -1 \\ b = 10 \end{array}$$

$\Rightarrow \quad y = -x^2 + 10x$

a) $S\left(-\frac{b}{2a} \Big/ \frac{4ac - b^2}{4a}\right) = \left(5/25\right)$

$\underline{\underline{h_{max} = 25\,m}}$

b) Nullstellen $\quad 0 = -x^2 + 10x$

$0 = x(-x + 10) \Rightarrow x_1 = 0$

$\Rightarrow x_2 = 10$

$\underline{\underline{\text{Entfernung } \Delta x = 10\,m}}$

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